PREPROCESSING IN SAT-BASED MULTI-OBJECTIVE OPTIMIZATION

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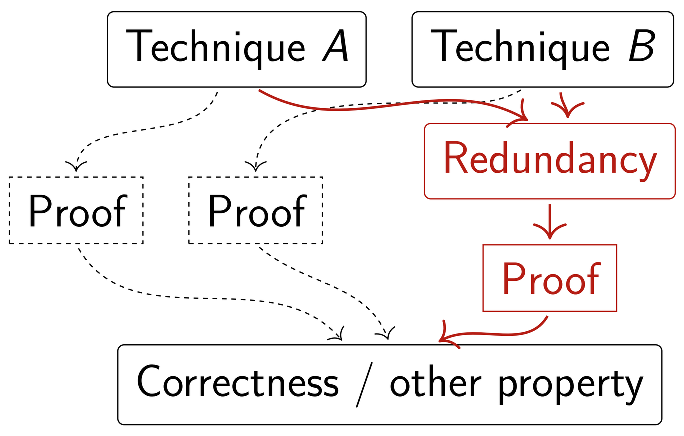
MOTIVATION

Maximum satisfiability (MaxSAT) solving has proven to be efficient for many real-world optimization problems

Many real-world applications have multiple conflicting objectives, calling for multi-objective **(MO-)MaxSAT**

Preprocessing (simplifying the instance before solving) is central in SAT, and becoming more popular in MaxSAT solving

WHY REDUNDANCY?



Instead of individual proofs (⇢), uniform proofs of correctness and analysis of power (→)

CONTRIBUTIONS

- 3 distinct redundancy notions
- Lifting (Max)SAT preprocessing techniques to MO-MaxSAT
- Open-source preprocessor for MO-MaxSAT: MaxPre 2.1
- Empirical evaluation

BACKGROUND

MULTI-OBJECTIVE MAXSAT

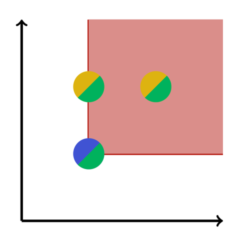
MO-MAXSAT INSTANCE

- Constraints: clauses
- Min. objectives: pseudo-boolean $O_i = \sum_x c_x^i \cdot x$

DOMINATING SOLUTIONS

τ **weakly dominates** δ: all objectives $O_i(\tau) \leq O_i(\delta)$
τ **dominates** δ: additionally one objective $O_i(\tau) < O_i(\delta)$

Every non-dominated solution is **Pareto-optimal**



AIM

Compute the non-dominated set (Pareto-optimal costs) ⇒ Preprocessing needs to preserve the **non-dominated set**

CONTRIBUTIONS

REDUNDANCY IN MO-MAXSAT

Redundant clause C_{red}: does not change the non-dominated set.

For every solution τ that does not satisfy C_{red}, there is a solution δ that weakly dominates τ and satisfies C_{red}.

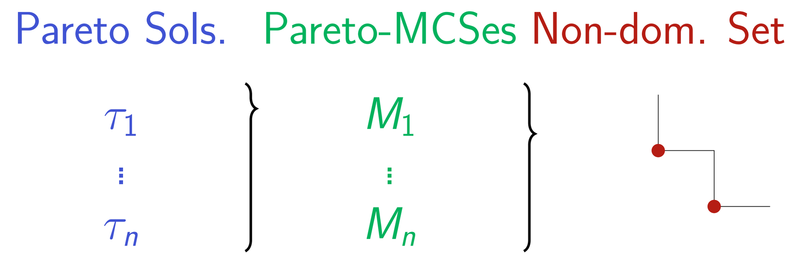
Reconstructible clause C_{rec}: redundant clause satisfied by forcing a fixed set of literals ω.

The weakly dominating solution is always $\delta = (\tau \setminus \neg\omega) \cup \omega$.

Literal-reconstructible clause: reconstructible clause with a single literal in the set ω.

PARETO-MINIMAL CORRECTION SETS

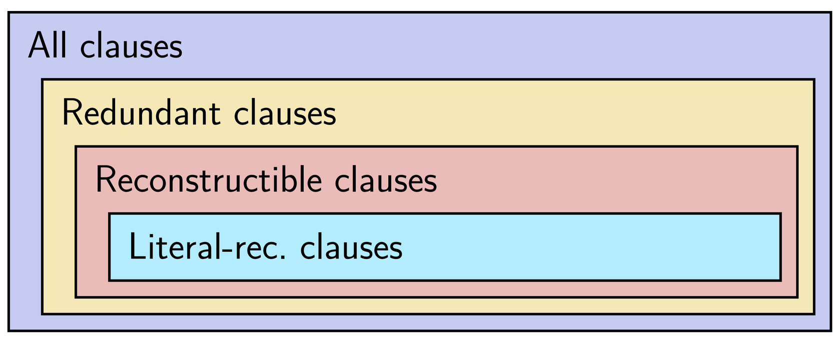
Pareto-MCS: multi-objective extension of minimal correction set.



At least one Pareto-MCS per non-dominated cost tuple, but not all need to be preserved.

Literal-reconstructible clauses preserve Pareto-MCSes, but reconstructible clauses do not.

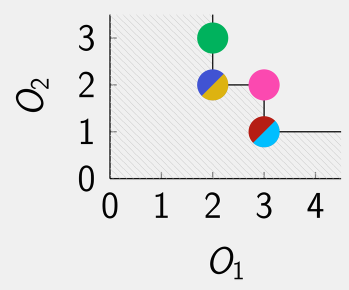
RELATIONSHIPS BETWEEN REDUNDANCY NOTIONS



EXAMPLE (REDUNDANT CLAUSES)

$$
\begin{aligned}
F &= \{(a_1 \lor a_2), (b_1 \lor b_2), (a_1 \lor b_1) \\
&\quad (a_1 \lor b_2), (a_2 \lor b_1), (a_2 \lor b_2), (a_3 \lor a_4)\} \\
O_1 &= a_1 + a_2 + a_3 + a_4 \\
O_2 &= b_1 + b_2 + b_3 \\
C_{\text{red}} &= (\neg a_2 \lor \neg b_2) \\
C_{\text{rec}} &= (\neg a_4), \quad \omega = \{a_3 \lor \neg a_4\} \\
C_{\text{l-rec}} &= (\neg b_3), \quad l = \neg b_3
\end{aligned}
$$

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	C_{red}	C_{rec}	$C_{\text{l-rec}}$
τ_1	1	0	1	0	1	1	0	1	1	1
τ_2	0	1	1	0	1	1	0	0	1	1
τ_3	1	1	1	0	1	0	0	1	1	1
τ_4	1	1	0	1	1	0	0	1	0	1
τ_5	1	1	1	1	1	1	0	0	0	1
τ_6	1	0	1	0	1	1	1	1	1	0



MAXPRE 2.1 SUPPORTS

LITERAL-RECONSTRUCTIBLE

- Bounded variable elimination
- Blocked clause elimination
- Subsumption elimination
- Unit propagation*
- Self-subsuming resolution
- Failed literal elimination*
- Equivalent literal substitution*
- TrimMaxSAT

*on non-objective literals

RECONSTRUCTIBLE

- (Group-)subsumed label elimination

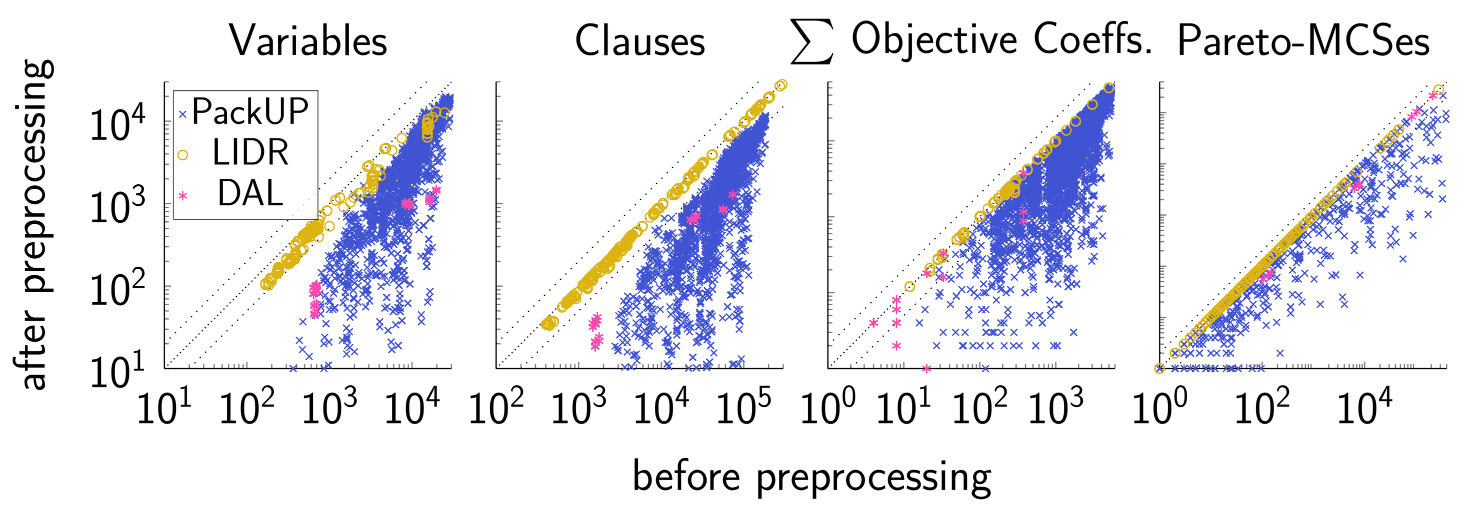
CHANGE OBJECTIVES

- Unit propagation+
- Equivalent literal substitution+
- Intrinsic at-most-ones
- Binary core removal

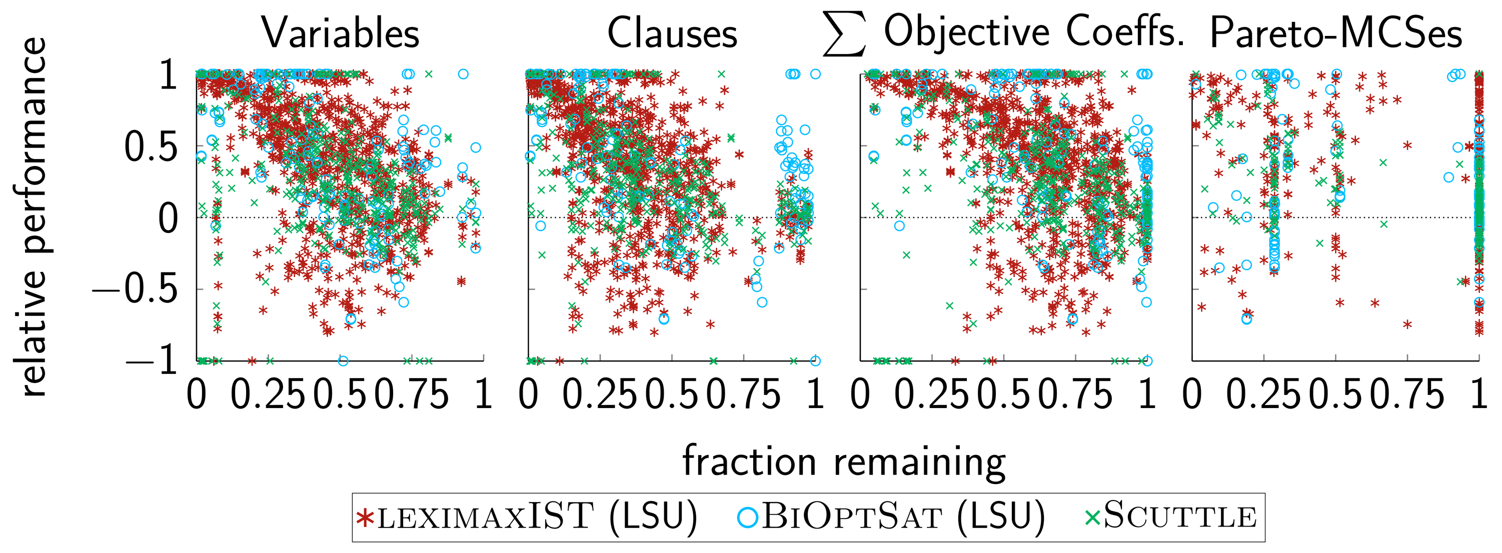
+on objective literals

EMPIRICAL EVALUATION

INSTANCE SIZE REDUCTION



IMPACT ON PER-INSTANCE SOLVER PERFORMANCE



IMPACT ON SOLVER PERFORMANCE

	PackUP		LIDR		DAL	
# inst.	3692 (1420*)		366		96	
Solver	Δ #	$\Delta \sum t$	Δ #	$\Delta \sum t$.	Δ #	$\Delta \sum t$
BIOPTSAT (bi-objective optimization)						
LSU	+27	−13.7	+3	−18.3	–	–
CG	+5	−6.4	±0	−5.5	–	–
Hybrid	+5	−13.6	±0	−4.9	–	–
SCUTTLE	+6	−40.4	−1	+7.2	+1	−0.6
CLM						
CG	−5	+14.5	±0	+1.4	−7	+4.7
IHS	−19	−68.7	−19	−3.3	−7	−0.1
LEXIMAXIST (leximax optimization)						
LSU	+71	−166.1	±0	+1.2	±0	+0.9
CG	+3	−0.7	+2	−5.7	+1	−3.2

Δ #: difference in number of solved instances

$\Delta \sum t$: difference in cumulative runtime over solved instances in 10^3 seconds

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