MaxSAT-Based Bi-Objective Boolean Optimization

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Constraint Reasoning and Optimization Research Group

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UNIVERSITY OF HELSINKI

INTRODUCTION

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Paper under review at international scientific conference (SAT'22)

1. Motivation
2. Contribution (BIOPTSAT)
3. Results
4. Take away points

MOTIVATION — OPTIMIZATION

Task: Choose the cheapest flat with at least two rooms

What if...

... we want to take commute into consideration as well?

Flat	Rooms	Price
A	2	300 000 €
B	2	240 000 €
C	1	180 000 €
D	2	270 000 €

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Task: Choose a flat with at least two rooms that is as cheap as possible while also having as short of a commute as possible.

Optimality for Multiple Objectives

There is no single definition of optimality for multiple objectives!

Pareto Optimality

All solutions for which no other solution is clearly better are considered optimal

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Task: From all valid decision trees, find the smallest ones that also minimize classification error

Hard Problems

Implicit solution definitions are $\mathcal{NP}$-hard for many real-world problems

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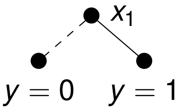

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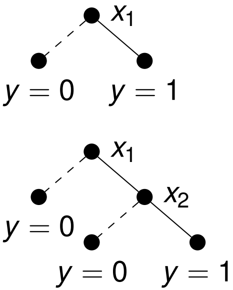

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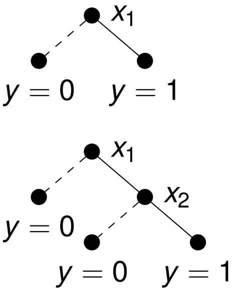

THESIS CONTRIBUTION

- Development of the BIOPTSAT algorithm
  - Enumeration of exact Pareto-optimal solutions to hard problems
- Implementation of the BIOPTSAT algorithm and its competitors (will be released open source)
- Evaluation of variants of BIOPTSAT and comparison to two competitors
- Evaluation of refinements to BIOPTSAT

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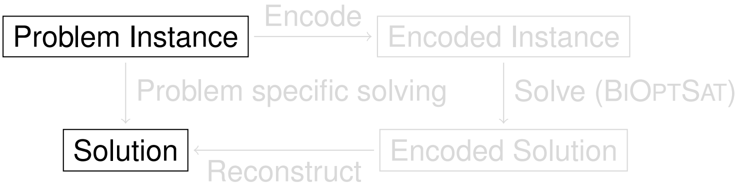
High-Level Overview of BiOptSat



- Encoding language: Boolean logic
- Lexicographic method
- Based on MaxSAT algorithms
- Making use of so-called SAT solver

High-Level Overview of BiOptSat

$$\begin{tikzcd} \text{Problem Instance} \arrow[r, "\text{Encode}"] \arrow[d, "\text{Problem specific solving}"] & \text{Encoded Instance} \arrow[d, "\text{Solve (BiOptSat)}"] \\ \text{Solution} & \text{Encoded Solution} \arrow[l, "\text{Reconstruct}"'] \end{tikzcd}$$

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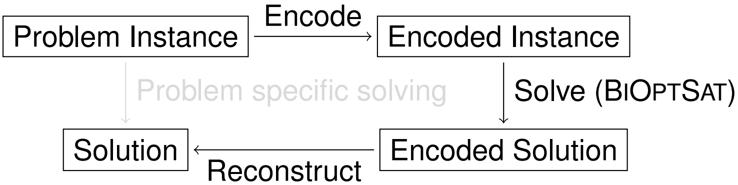

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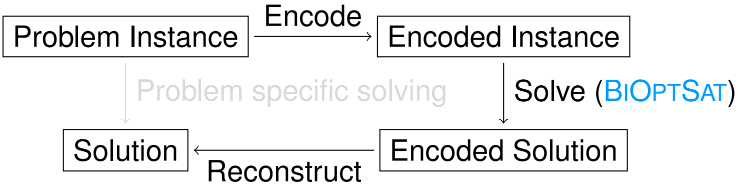

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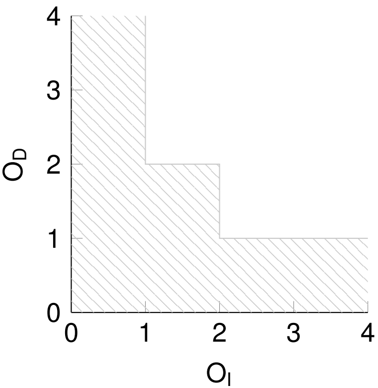
Search Progression of BiOptSat



- Ordered enumeration from top left to bottom right

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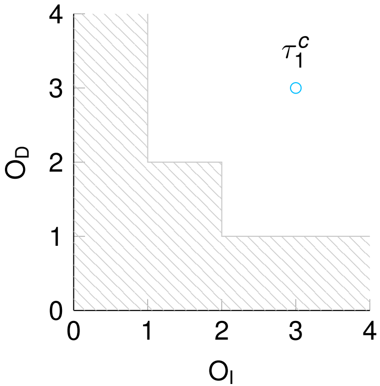

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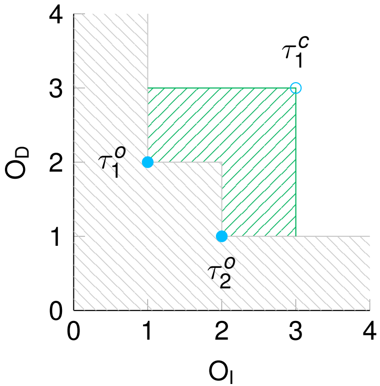

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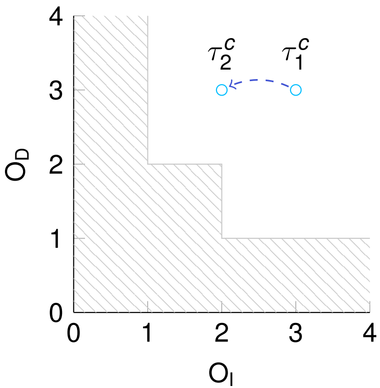

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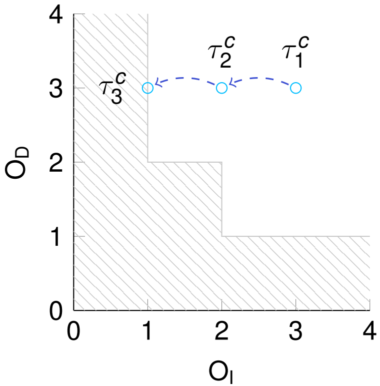

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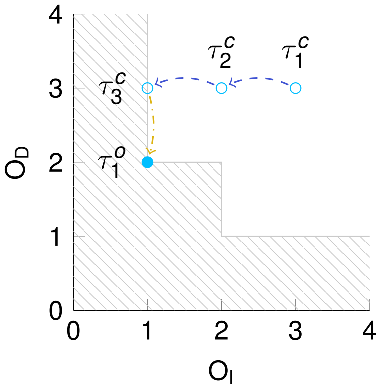

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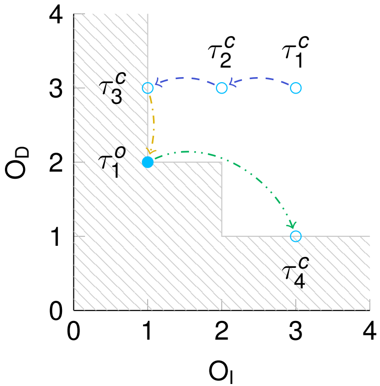

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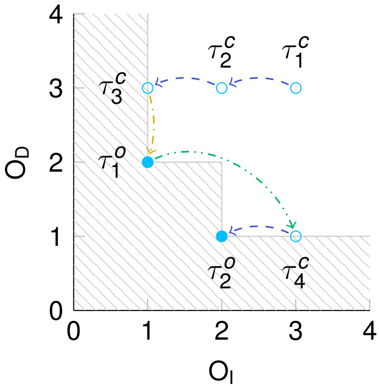

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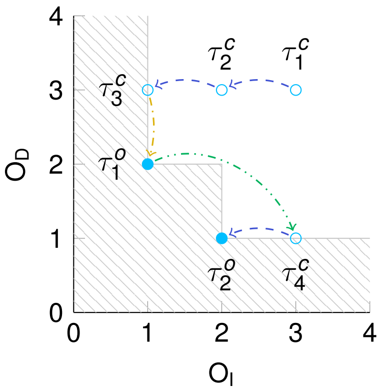

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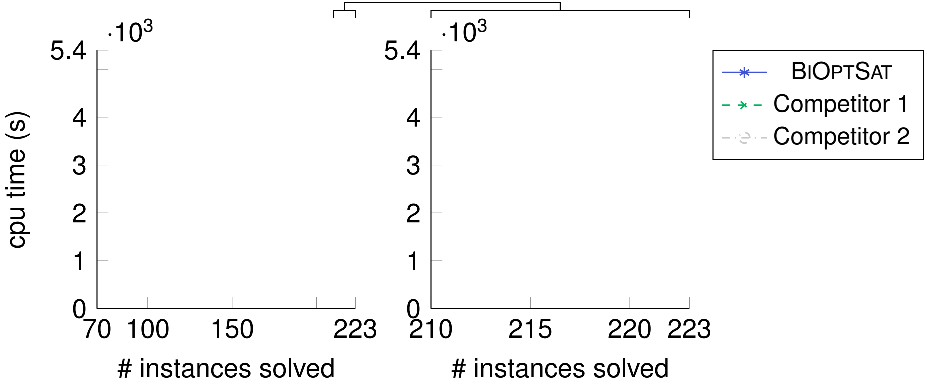
RESULTS — SOLVED INSTANCES

Benchmark: Learning Pareto-optimal decision rules



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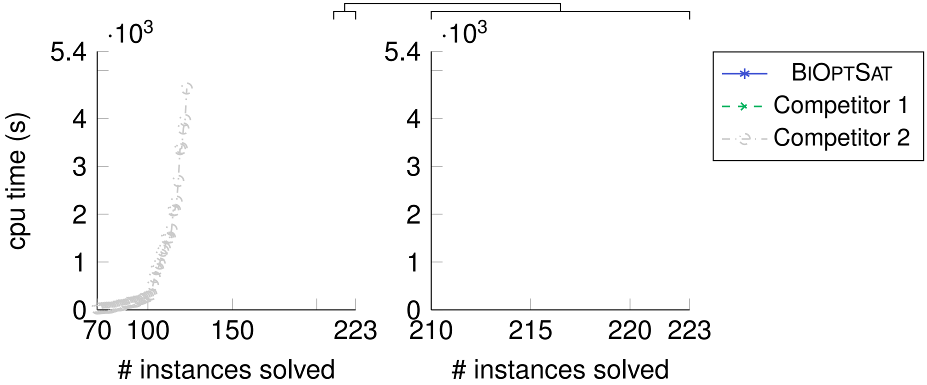

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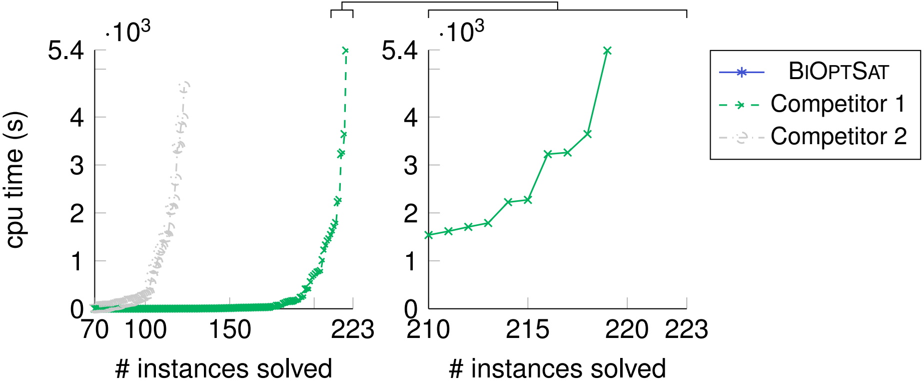

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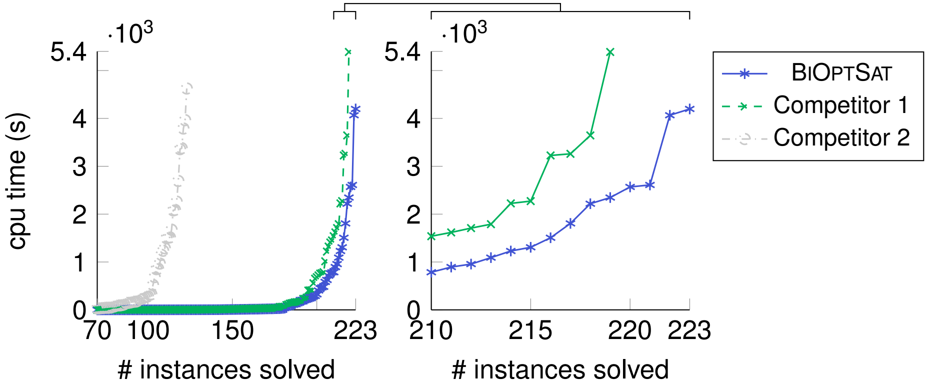
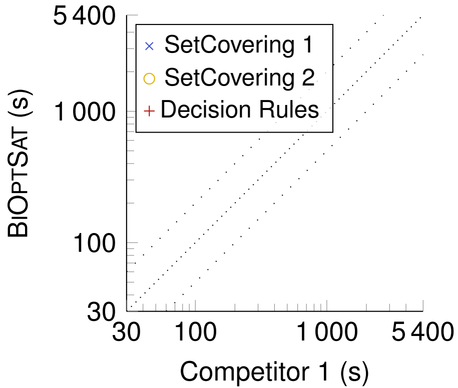


Results — Instance Runtimes



- BiOptSat outperformed both competitors
- Amount of outperforming competitors depends on benchmark

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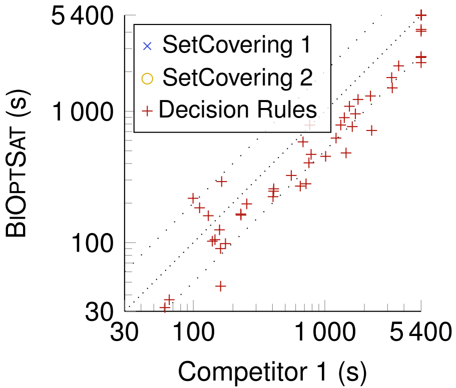

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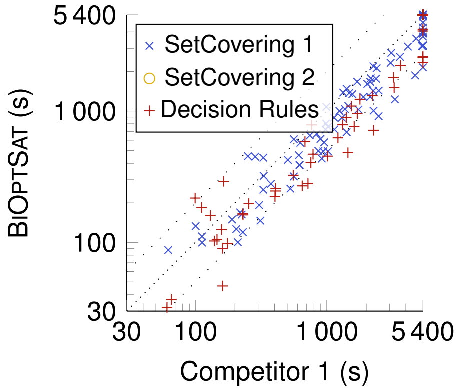

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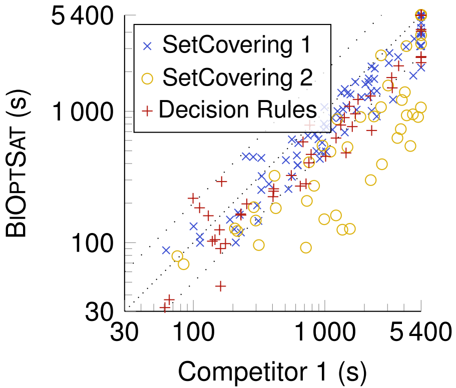

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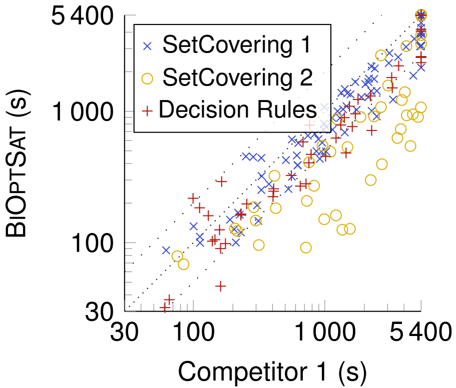

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Take Away Points

Content of Thesis

- Paper in review: significant result for a thesis project
- (Exact) bi-objective optimization is interesting and not that well researched

Thesis Process

- Starting with a summer internship is great
- Working as a research assistant allows for diving deep into a topic

Thank you for your attention!

Pareto Optimality

Definition (Domination)

Given two objective functions O_1, O_2 and two solutions τ, τ', x dominates τ' if (i) $O_i(\tau) \leq O_i(\tau')$ for all $i \in \{1, 2\}$, and (ii) $O_i(\tau) < O_i(\tau')$ for some $i \in \{1, 2\}$. We represent τ dominating τ' as $\tau \prec \tau'$.

Definition (Pareto optimality)

A solution τ is Pareto-optimal iff there is no τ' such that $\tau' \prec \tau$, i.e., τ is not dominated by any other solution.

BiOptSat

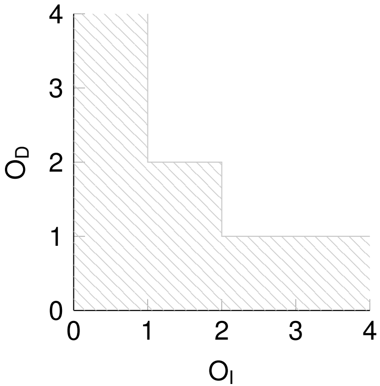
Algorithm 1 BiOptSat: MaxSAT-based bi-objective optimization

Input: A formula F, two objectives O_I and O_D.

Output: Either one or all Pareto-optimal solution for each Pareto point of F.

1: $\tau^r \leftarrow \texttt{InitSATsolverAndSolve}(F)$ {Invokes the SAT solver on the formula.}
2: $b_D \leftarrow \infty, b_I \leftarrow 0$
3: **while** res $=$ SAT **do**
4: $(b_I, \tau^r) \leftarrow \texttt{Minimize-Inc}(b_D, O_I(\tau^r))$ {Maintains Tot(O_I) (or similar)}
5: $(b_D, \tau^r) \leftarrow \texttt{Solution-Improving-Search}(b_I, O_D(\tau^r))$ {Builds Tot(O_D)}
6: **yield** τ^r {Optionally: **yield** $\texttt{EnumSols}(b_D, b_I)$}
7: $(\text{res}, \tau^r) \leftarrow \texttt{isSAT}(\{\langle O_D < b_D \rangle\})$

Search Details of BiOptSat



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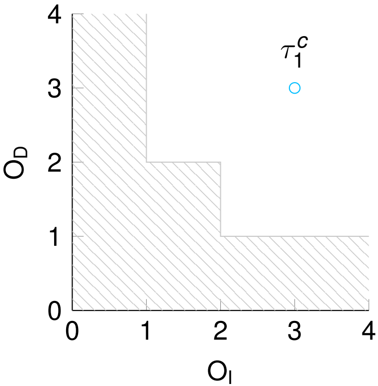

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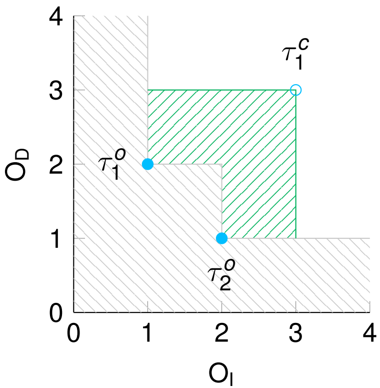

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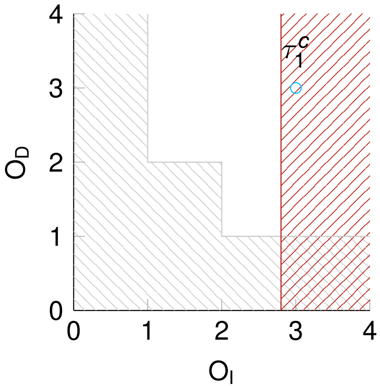

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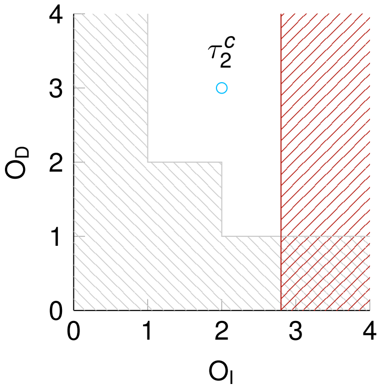

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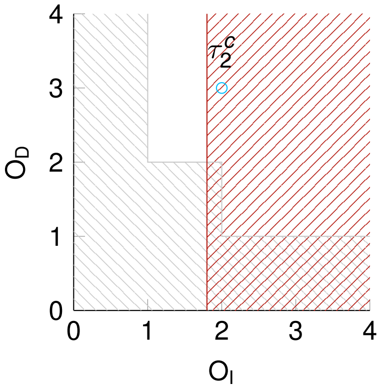

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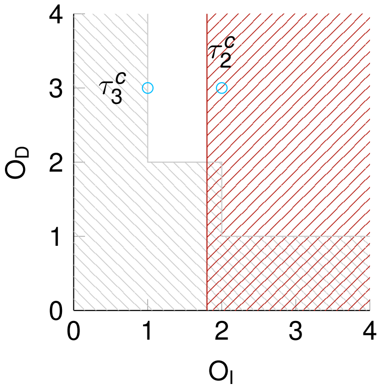

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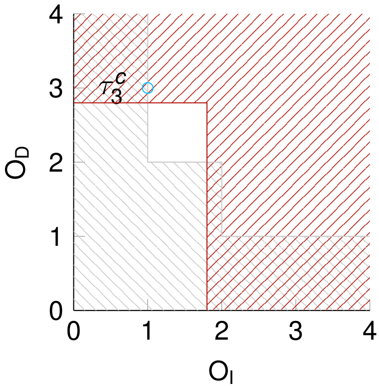

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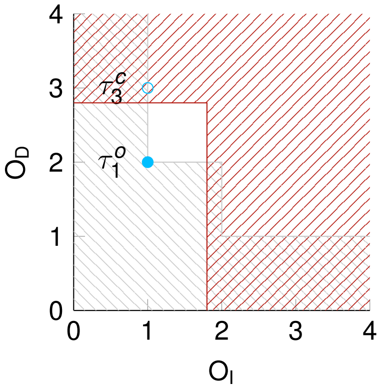

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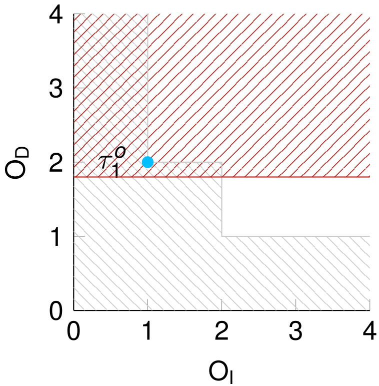

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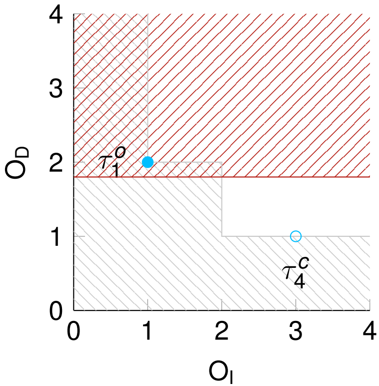

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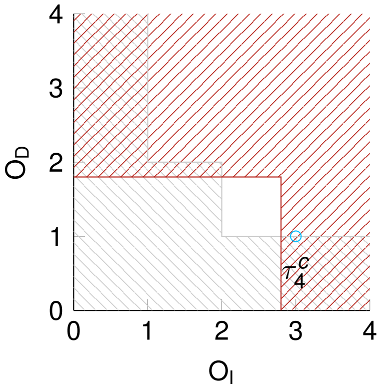

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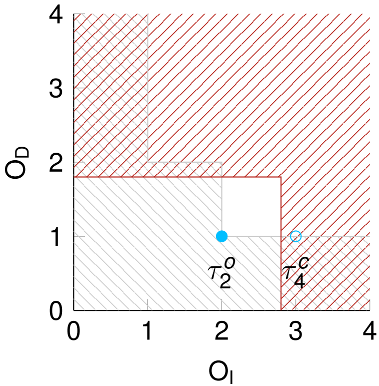

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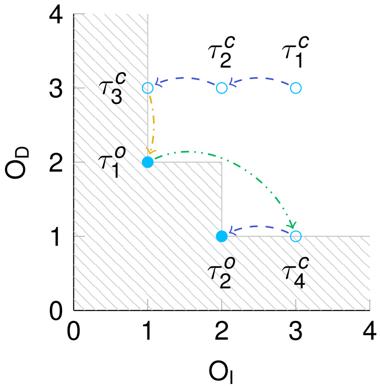

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