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Certifying Pareto Optimality in Multi-Objective Maximum Satisfiability

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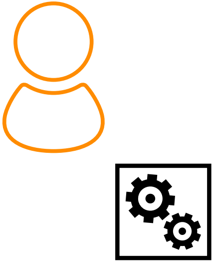
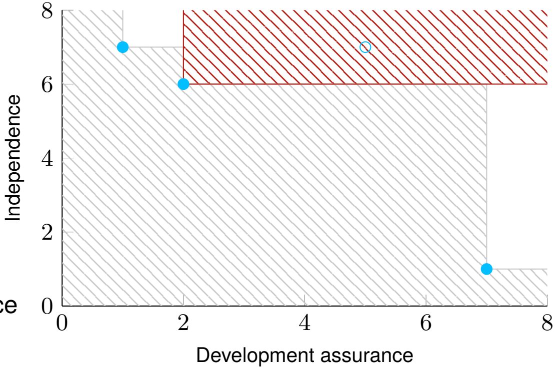
Multi-Objective Optimization

How to deal with conflicting objectives

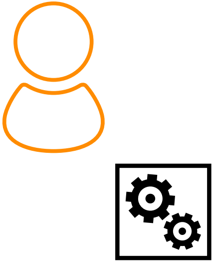




- Aviation: assurance level vs. independence
- Decision tree: accuracy vs. intrepretability

The Problem

Can you trust my solver?



- Aviation: assurance level vs. independence
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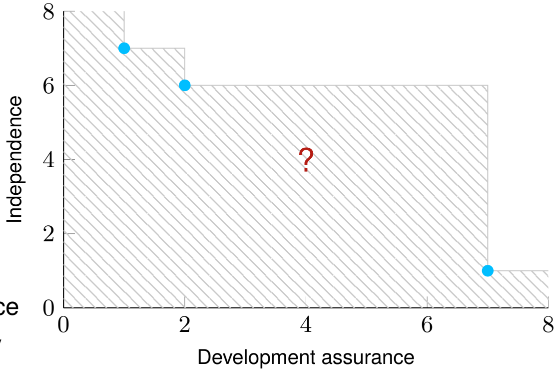
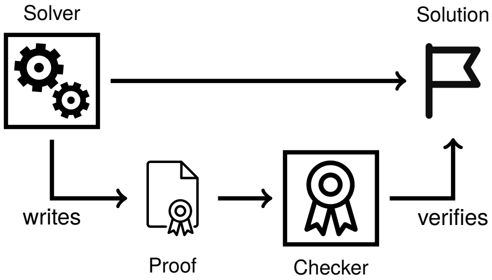


Proof Logging

If we can't trust the solver, can we trust the result?

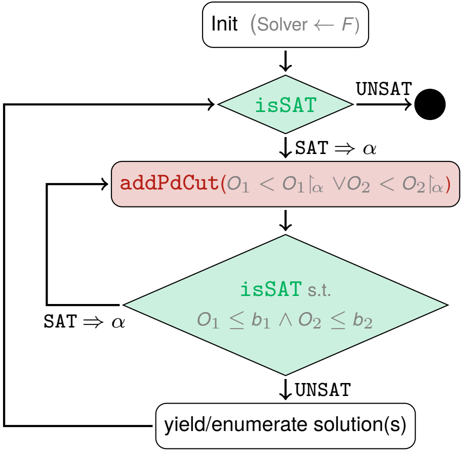


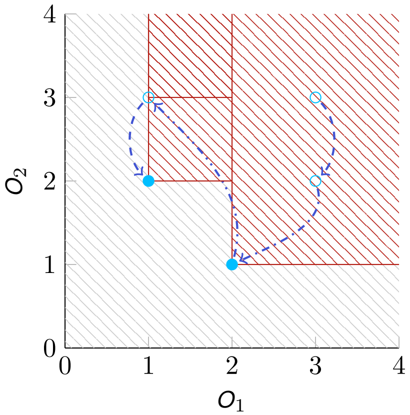
Paradigm	Proof format
SAT	DRAT
MaxSAT	VeriPB
MO-MaxSAT	This talk

The *P*-Minimal Algorithm

Multi-objective solution-improving search

[Soh et al. 2017]





The VeriPB Proof System

Pseudo-Boolean cutting planes proofs

[Bogaerts et al. 2023]

- Pseudo-Boolean Constraints
- Derive constraints by linear combination
- Redundant constraints
- Exclude solutions after finding them
- Single-objective optimization
- Preorder for expressing preference

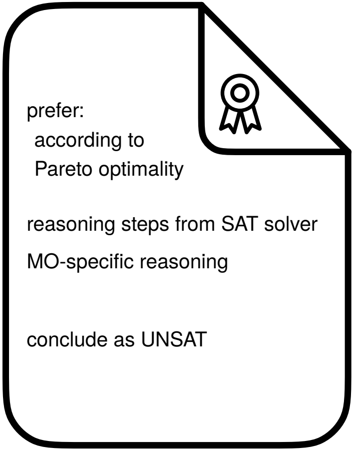
minimize:
$1x_1 + 2x_2 + 1x_3$

prefer:
$\overline{x_4}$ over x_4

[A]	$3x_1 + 2x_2 + 1x_3 \geq 3$
[B]	$\overline{x_1} + x_4 \geq 1$
[A+3B]	$2x_2 + 1x_3 + 3x_4 \geq 3$
[red]	$1x_1 + 1x_2 + 2x_5 \geq 2$
~~[red]~~	~~$1x_4 \geq 1$~~
[solx]	exclude x_1, x_2, x_3, x_4, x_5
[soli]	better than $x_1, x_2, x_3, x_4, \overline{x_5}$

Multi-Objective Proof Setup

What VeriPB can already give us



Guarantee

For each non-dominated point at least one solution explicitly appears in the proof

Syntactic restrictions

- First step in proof must load the Pareto order
- Order must never be changed

Encoding Pareto Dominance as a VeriPB Order

Telling VeriPB about the objectives

Given $O_1, \ldots, O_p$

Required VeriPB order:

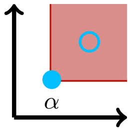
formula that is true iff α (weakly) dominates β

VeriPB Pareto order

$$O_i\restriction_\alpha \leq O_i\restriction_\beta, \quad \text{for } i = 1, \ldots, p$$

Certifying Pareto Dominance Cuts

The building block for all algorithms



$$O_1 < O_1\restriction_\alpha \vee O_2 < O_2\restriction_\alpha$$

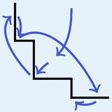
1. Reified objective CNF encoding
$$w_1 \Leftrightarrow O_1 < O_1\restriction_\alpha$$
2. Map each weakly dominated solution to α
(Redundant with α as witness)
3. Exclude α itself
4. Derive PD cut by combining previous two constraints

Proof Logging MO-MaxSAT Algorithms

Putting everything together

P-Minimal

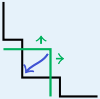
[Soh et al. 2017]



- SAT solver reasoning
- CNF objective encodings
- PD cuts

Lower-Bounding

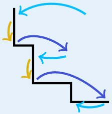
[Cortes et al. 2023]



Upper-bounds irrelevant
→ same as *P*-Minimal

BiOptSat

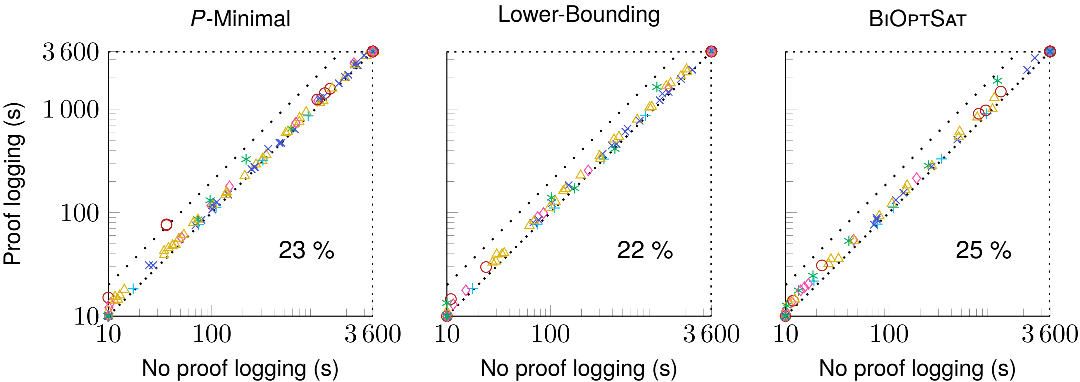
[Jabs et al. 2024]



- Derive lower-bound on first objective
- Certify PD cut
- Strengthen PD cut based on known lower-bound

Proof Logging Overhead

How expensive is this



Proof Logging for Multi-Objective MaxSAT

Summary and conclusions

- MO-MaxSAT certificates that all non-dominated points were discovered
- Proofs in VeriPB format without extension
- Low overhead for proof logging
- Open-source implementation

Paper, slides, code, and contact:

`christophjabs.info/tacas25`

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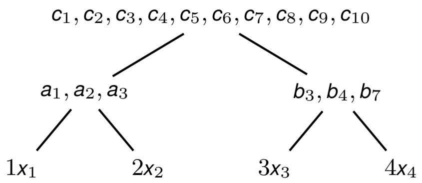
Certifying the Generalized Totalizer Encoding

[Vandesande et al. 2022]

Certified reified pseudo-Boolean CNF encodings

$$O = 1x_1 + 2x_2 + 3x_3 + 4x_4$$



1. Semantic definition of variables

$$a_2 \Leftrightarrow 1x_1 + 2x_2 \geq 2$$

(redundant because fresh variable)

2. Derive clauses from semantic definitions

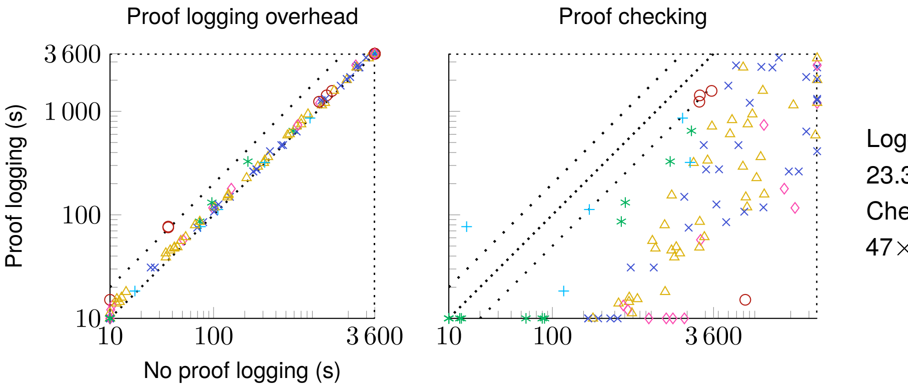
$(a_2^{\Rightarrow} + b_3^{\Rightarrow} + c_5^{\Leftarrow} := (\overline{a_2} \vee \overline{b_3} \vee c_5))$

Our contribution

- This also works for the *generalized* totalizer
- Adapting process for complex core-boosted encodings

Proof Checking Performance

VeriPB still needs a lot of optimization work



Logging overhead: 23.3%

Checking: 47× longer