Bi-Objective MaxSAT and the BIOPTSAT Algorithm

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What?

Bi-Objective MaxSAT

- NP-hard bi-objective optimization
- Notions of optimality

P-Minimal

For comparison [Soh et al. 2017]

BiOptSat Algorithm

- Enumeration of Pareto-optimal solutions
- Using ideas from MaxSAT solving
- 5 variants

Links to all materials:

- `christophjabs.info/cph22`

So What?

Applications for Bi-Optimization

- Interpretable classifiers [Malioutov and Meel 2018]
- Solver portfolios [Janota et al. 2021]
- Network routing [Silvério et al. 2022]
- Supply chain optimization [Pinto-Varela et al. 2011]

Scalarizing is not enough

$$O = O_1 + \lambda O_2$$

Conclusion

Need for SAT-based algorithms for bi-objective optimization

Example — Interpretable Decision Rules

Task: Find decision rules that minimize size and classification error

Definition (Decision Rule)

Propositional formula over binary features. Result of formula represents binary classification.

Pareto-optimal decision rules

Dataset

x_1	x_2	y
1	1	1
0	1	0
1	0	0

Decision rules

$$r_1 = x_1 \land x_2$$

$$r_2 = x_1$$

Single- vs. Multi-Objective Optimization

Single-Objective Optimization

- Unique optimal cost value
- Possibly multiple optimal solutions

Multi-Objective Optimization

- What even is optimality?

MO Notions of Optimality

- Lexicographic optimality
  (Optimize in order of importance / Lexicographically compare solutions)
- Leximax optimality
  (Optimize worst objective first / *Sort* and lexicographically compare)
- Pareto optimality
  (Every solution without an *obviously* better one is optimal)

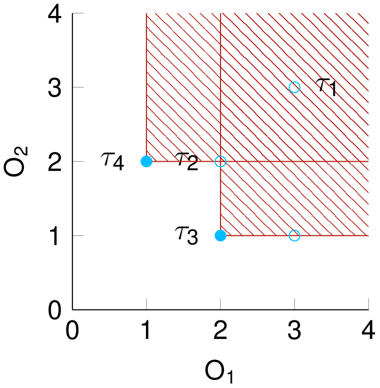
Pareto Optimality

Definition (Dominance)

A solution dominates another one if it is *not worse* on all objectives and *better* on at least one.

Definition (Pareto Optimality)

Every solution that is not dominated by any other solution is Pareto-optimal.



The Problem

Definition (Bi-Objective MaxSAT)

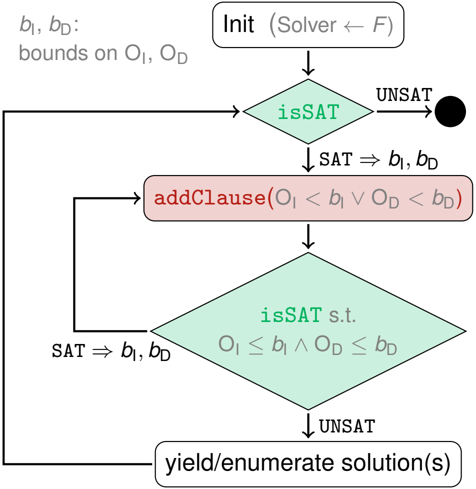
- Hard clauses F
  (SAT encoding of the constraints)
- 2 linear PBO objectives O_I, O_D
  (Weighted relaxation literals)

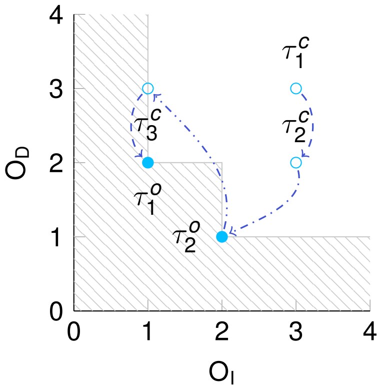
Toy Instance

$$F = \texttt{As-CNF}\left(\sum_{l \in O_I \cup O_D} \tau(l) \geq 3\right),$$
$$O_I = i_1 + i_2 + i_3,$$
$$O_D = d_1 + d_2 + d_3$$

P-Minimal: "Multi-Objective Solution-Improving Search" [Soh et al. 2017]

b_I, b_D: bounds on O_I, O_D

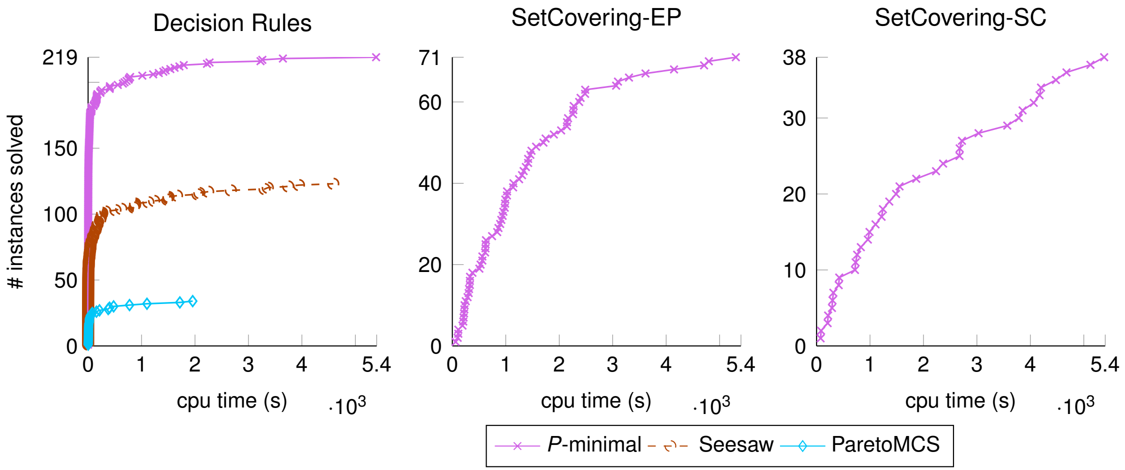




P-Minimal: Implementation Details

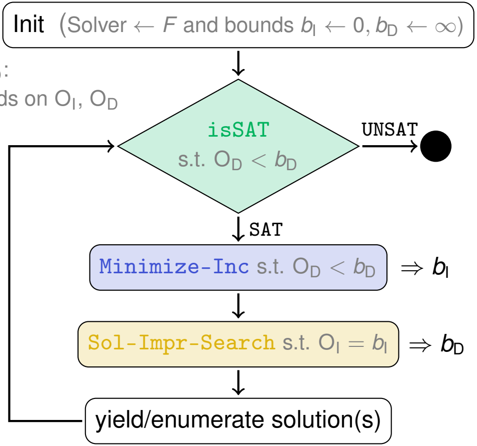
- Single SAT solver instance (CaDiCaL [Biere et al. 2020])
- C++ implementation
- Instead of order encoding as in [Soh et al. 2017], totalizers [Bailleux and Boufkhad 2003]

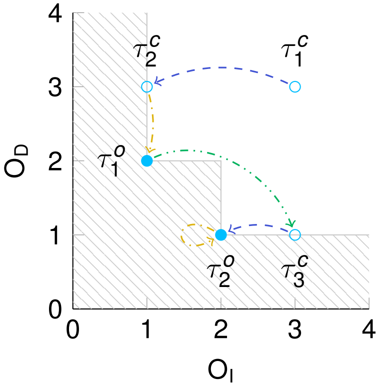
P-Minimal: Results



BiOptSat: The Lexicographic Method

b_I, b_D:
bounds on O_I, O_D





BIOPTSAT: Implementation Details

- Single SAT solver instance (CaDiCaL [Biere et al. 2020])
- Heavy use of incremental totalizers [Martins et al. 2014]
- C++ implementation

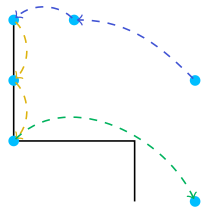
BiOptSat SAT-UNSAT

Simplest variant of Minimize-Inc:

SAT-UNSAT

- Totalizer over objective literals
- Enforce $O_l < b_l$ with assumptions until UNSAT

BiOptSat SAT-UNSAT



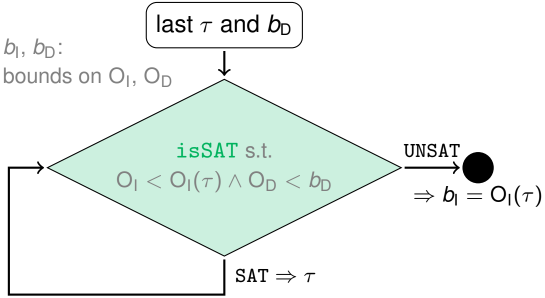
Sol-Impr-Search always solution improving search

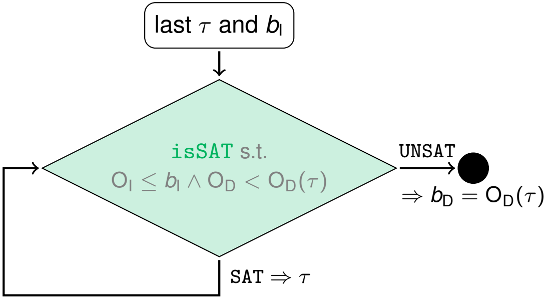
- Lower-bounding impractical because of decreasing objective values
- Loosening of constraints on O_l can invalidate cores
- Totalizer over all objective literals

BiOptSat SAT-UNSAT: Details

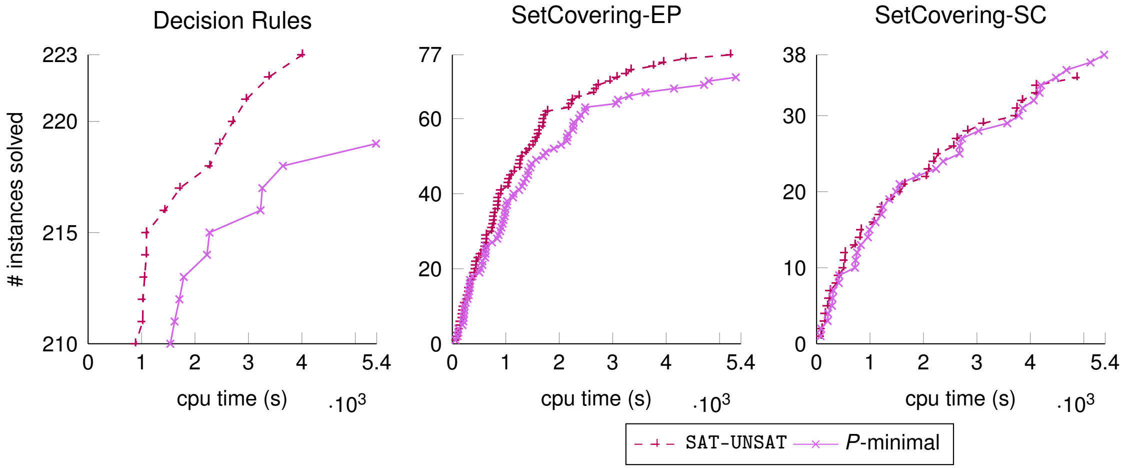
Minimize-Inc (SAT-UNSAT)

Sol-Impr-Search





BiOptSat SAT-UNSAT: Results



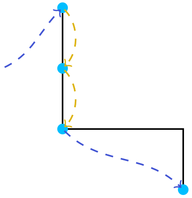
BiOptSat Lower-Bounding

Lower-bounding Minimize-Inc

- Incrementally built totalizer over objective literals
- UNSAT-SAT: increase bound until SAT
- MSU3 & OLL: core guided search
- Next iteration, continue from last found optimum

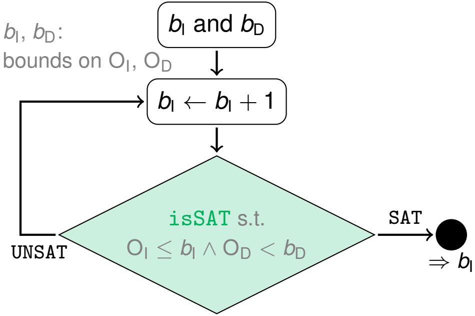
Sol-Impr-Search unchanged

BiOptSat UNSAT-SAT / MSU3 / OLL



BiOptSat Lower-Bouning: Details

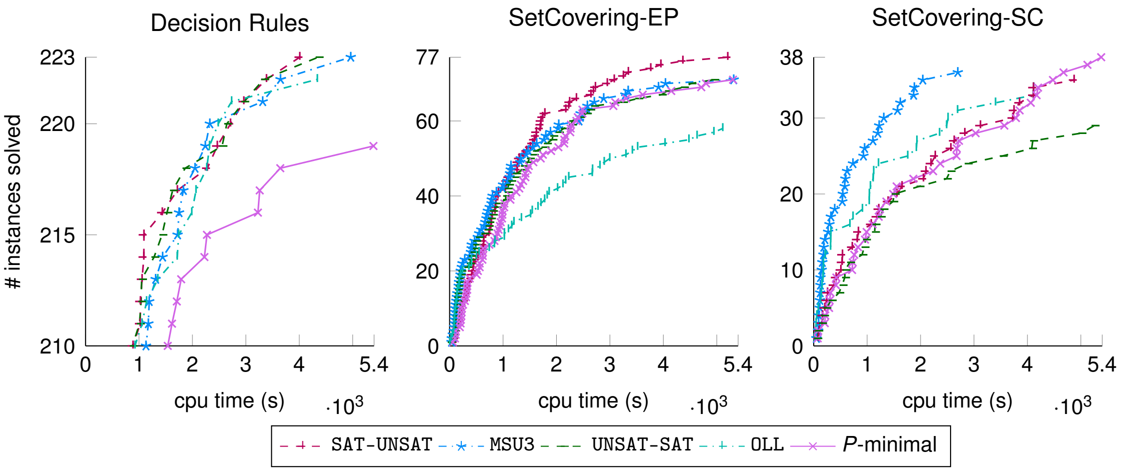
`Minimize-Inc` (UNSAT-SAT)



`Minimize-Inc` (MSU3)

(res, $\tau, \kappa) \leftarrow$ `isSAT`$(\{\langle O_D < b_D \rangle, \langle O_I \leq b_I \rangle\} \cup \{\neg l \mid l \in O_I \setminus \texttt{Act}\})$
while res = UNSAT **do**
  $b_I \leftarrow b_I + 1$
  $\kappa \leftarrow \kappa \setminus \{\neg\langle O_D < b_D \rangle, \neg\langle O_I \leq b_I \rangle\}$
  $\texttt{Act} \leftarrow \texttt{Act} \cup \kappa$
  build or extend Tot$(\texttt{Act}, b_I)$
  (res, $\tau, \kappa) \leftarrow$ `isSAT`$(\{\langle O_D < b_D \rangle, \langle O_I \leq b_I \rangle\} \cup \{\neg l \mid l \in O_I \setminus \texttt{Act}\})$
return b_I

BiOptSat Lower-Bounding: Results



MSHybrid: Avoiding UNSAT-SAT in MSU3

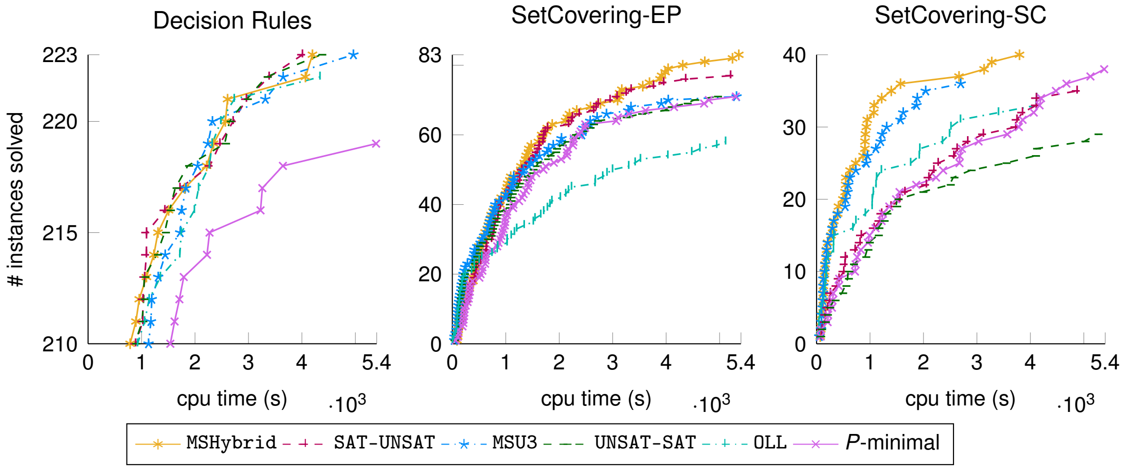
Search in Minimize-Inc continues after first optimum

- Often all literals active (core-guided variants)
- MSU3 with all literals active = UNSAT-SAT
- Performance suffers
- ⇒ Switch to SAT-UNSAT

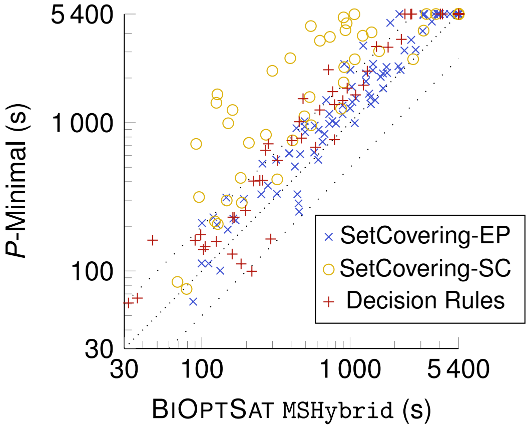
MSHybrid

1: **if** $|\texttt{Act}| < \texttt{thr} \cdot |O_I|$ **then**
2: MSU3
3: **else**
4: SAT-UNSAT

BiOptSat MSHybrid: Results



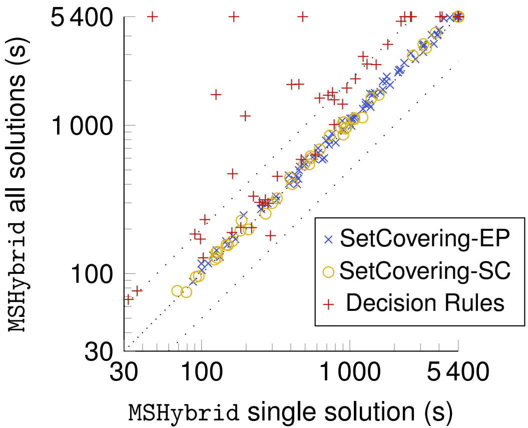
BiOptSat vs. *P*-Minimal



- BiOptSat restricts the SAT solver more
- This restriction allows for improving on the used algorithms
  - Allows lower-bounding (including core-guided) search
  - Hardening of the decreasing objective
- Most of the advantages of BiOptSat limited to two objectives

Enumeration of *All* Pareto-Optimal Solutions

- Every cost point can have multiple solutions associated with it
- Both BiOptSat and *P*-Minimal can enumerate all solutions
- Feasibility depends on the instance



Refinement: Domain Specific Blocking

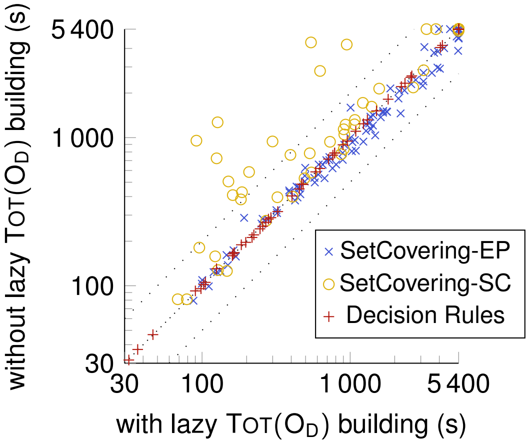
For enumeration of all Pareto-optimal solutions, blocking clauses for solutions are needed

- Generally: block over all variables in F (or all decisions of the SAT solver)
- Improved blocking clauses using domain knowledge
  - Shorter clauses (omitting functionally defined variables)
  - Block multiple symmetric solutions at once (omitting auxiliary variables)

Refinement: Lazily Building Both Totalizers

One totalizer over each objective

- Incrementally build "increasing" totalizer
  [Martins et al. 2014]
- Core-guided variant and overlapping objectives
  ⇒ "Decreasing" totalizer can be incrementally built
  - If literal inactive in increasing objective, ignore for "decreasing" totalizer



Summary

- BiOptSat algorithm
  - Enumerate Pareto-optimal solutions of CNFs
- Open-source implementation
- Outperforms 3 key competitors
- Best-performing variant: `MSHybrid`

Paper, slides, code, and contact:

`christophjabs.info/cph22`

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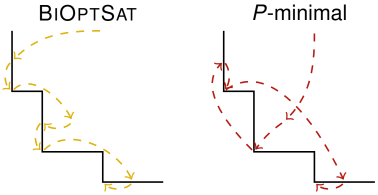
Competing Approaches

P-minimal [Soh et al. 2017]

- Closest to BiOptSat
- Unstructured order of enumeration



Seesaw [Janota et al. 2021]

- IHS framework generalized to two objectives
- Instantiating black box with solution improving search

ParetoMCS [Terra-Neves et al. 2018]

- Enumerate *all* MCSes and filter non-optimal solutions
- Available Sat4j implementation

Seesaw Instantiation

[Janota et al. 2021]

- Oracle: solution improving search in SAT solver
  - Minimal decision rule size to misclassify at most samples in hitting set
  - CaDiCaL [Biere et al. 2020]
- Cost function (hitting set extraction): MILP solver (CPLEX)
- (Optimization) core extraction: version for anti-monotone oracles in paper [Janota et al. 2021]

Number of Solved Instances

Instance Type	SAT-UNSAT		UNSAT-SAT		MSU3		OLL		MSHybrid		P-minimal	
	single	all	single	all	single	all	single	all	single	all	single	all
Decision Rules	**223**	**215**	**223**	**215**	**223**	**215**	222	213	**223**	**215**	219	213
SetCovering-EP	77	75	71	71	71	70	58	58	**83**	**81**	71	68
SetCovering-SC	35	35	29	29	36	36	34	34	**40**	**40**	38	26